Statistics

Summer 2023

Lecture 14

Feb 19-8:47 AM

$\boxed{58\% \text{ of}}$ $\boxed{172 \text{ Females}}$ randomly Selected were in Favor of changes in affirmative action.

Find 98% Conf. interval for the prop. of all Females in Favor of changes in affirmative action.

$\to$ C-level : .98

$n = 172$

$\hat{p} = .58$

$\Rightarrow x = n\hat{p} = 172(.58) = 99.76 \to \boxed{x = 100}$

if decimal $\to$ Round-up

1-PropZInt

$x = 100$

$n = 172$

C-level : .98

Calculate

$\boxed{.494 < p < .669}$

$49\% < p < 67\%$

we are 98% Confident that from 49% to 67% of all Females are in Favor of changes in affirmative action.

$E = \dfrac{.67 - .49}{2}$

$= \boxed{.09}$

$\hat{p} = \dfrac{.67 + .49}{2}$

$= \boxed{.58}$

Jul 6-7:33 AM

In a survey of 225 nurses, 100 of them were happy with work related issues.

Find conf. interval for the prop. of all nurses that feel the same.

$\boxed{.380 < P < .509}$

1-PropZInt
NO C-level
use .95

$n = 225$

$x = 100$

$\hat{p} = \frac{x}{n} = \frac{100}{225} \approx .444$

$38\% < P < 51\%$

$E = \frac{.509 - .380}{2} = \boxed{.065}$

$\hat{p} = \frac{.509 + .380}{2} = \boxed{.445}$

Due to rounding

Jul 6-7:42 AM

How to determine minimum sample size needed to construct confidence interval

1) Proportion

$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

with some algebra work

$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$

If decimal ⇒ Round-up

When $\hat{p}$ & $\hat{q}$ unknown use .5 for each

$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2$

Jul 6-7:49 AM

Find minimum Sample Size (n) needed to Construct 90% Conf. interval for pop. proportion with error not to exceed 5% and $\hat{p}=.6$

$\hat{p}=.6$

$\hat{q}=.4$

$E=.05$

$$n=\hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2=(.6)(.4)\left(\frac{1.645}{.05}\right)^2$$

$=259.7784$

$\approx \boxed{260}$

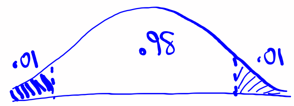
$Z_{\alpha/2}=$ invNorm(.95,0,1) $=1.645$

Suppose $\hat{p}$ & $\hat{q}$ were unknown and we wished 98% conf. interval.

$$n=.25\left(\frac{2.326}{.05}\right)^2$$

$=541.0276$

$\boxed{n \approx 542}$



$Z_{\alpha/2}=$ invNorm(.99,0,1)

$=2.326$

76% of 150 randomly Selected students were in favor of multiple-choice exams in math.

$n=150$

$\hat{p}=.76$

$\Rightarrow x=n\hat{p}=150(.76)=\boxed{114}$

Find 99% conf. interval for the prop. of all Students in favor of multiple-choice exams in math.

C-level: .99

1-PropZInt

$\boxed{.670<p<.850}$

$67\%<p<85\%$

$$E=\frac{.85-.67}{2}=\boxed{.09}$$

$$\hat{p}=\frac{.85+.67}{2}=\boxed{.76}$$

Find minimum Sample Size needed if we wish to be 99% confident and error not to exceed 5%.

$\hat{p} = .76$

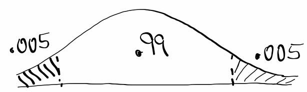
$\hat{q} = .24$

$E = .05$

$$n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2 = (.76)(.24)\left(\frac{2.576}{.05}\right)^2$$

$= 484.146$

$\boxed{n \approx 485}$



$Z_{\alpha/2} = \text{invNorm}(.995, 0, 1) = \boxed{2.576}$

Assume $\hat{p}$ & $\hat{q}$ are unknown, C-level not given and error is not to exceed 4%.

$\Downarrow$

use 95%

$$n = .25\left(\frac{Z_{\alpha/2}}{E}\right)^2 = .25\left(\frac{1.960}{.04}\right)^2 = 600.25$$

$\boxed{n \approx 601}$

$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

Jul 6-8:10 AM

Estimate Population Mean μ:

Point-estimate $\qquad < \mu <$

$$\bar{x} - E < \mu < \bar{x} + E$$

$\downarrow$

$\bar{x} \to$ Sample Mean, $E \to$ Margin of error

Case I: σ Known	Case II: σ Unknown
$E = Z_{\alpha/2} \cdot \dfrac{\sigma}{\sqrt{n}}$	$E = t_{\alpha/2} \cdot \dfrac{s}{\sqrt{n}}$ $\to df = n-1$
Z Interval inpt: $\boxed{\text{Stats}}$	T Interval inpt: $\boxed{\text{STATS}}$

Jul 5-10:03 AM

Given $n=36$, $\bar{x}=125$, $\sigma=20$, C-level: .92

Find Conf. interval for μ.

Since σ is known $\Rightarrow$ Z Interval

inpt: $\boxed{\text{Stats}}$

Since $\bar{x}$ is a whole # $\rightarrow$ $\boxed{119 < \mu < 131}$

$E = \dfrac{131 - 119}{2} = \boxed{6}$

$\bar{x} = \dfrac{131 + 119}{2} = \boxed{125}$

Jul 6-8:42 AM

Given: $n=18$, $\bar{x}=82.5$, $S=9.8$

Find Conf. interval for μ

No C-level $\Rightarrow$ Use .95

$\bar{x}$ is in 1-decimal $\rightarrow$ Round to 1-decimal

σ unknown $\rightarrow$ T Interval

inpt $\boxed{\text{Stats}}$

$\boxed{77.6 < \mu < 87.4}$

$E = \dfrac{87.4 - 77.6}{2} = \boxed{4.9}$

$\bar{x} = \dfrac{87.4 + 77.6}{2} = \boxed{82.5}$

Jul 6-8:47 AM

40 randomly Selected teachers in LAUSD had a mean age of 38.5 Yrs.

$n=40$

$\bar{x}=38.5$

C-level: .9

Find 90% Conf. interval for the mean age of all teachers in LAUSD.

$\boxed{36.0 < \mu < 41.0}$

Assuming $\boxed{\text{Standard deviation}}$ of ages of $\boxed{\text{all}}$ teachers in LAUSD is $\boxed{9.75}$ Yrs. $\sigma=9.75$

Since σ Known → Z Interval

Since $\bar{x}$ is 1-decimal → Round to 1-decimal

$E=\dfrac{41-36}{2}=2.5$

$\bar{x}=\dfrac{41+36}{2}=38.5$

Z Interval

inpt: $\boxed{\text{Stats}}$

$\sigma=9.75$

$\bar{x}=38.5$

$n=40$

C-level: .9

Jul 6-8:51 AM

12 QZ were randomly Selected.

Here are the Scores

10	15	18	20
8	10	12	10
15	16	20	14

Find

$\bar{x}=14$

$S=4$

Round to whole #

Construct Conf. interval for the mean of all QZ.

$\boxed{11 < \mu < 17}$

NO C-level ⇒ .95

σ Unknown ⇒ TInterval

$\bar{x}$ is whole # ⇒ Round to whole #

$E=\dfrac{17-11}{2}=3$

$\bar{x}=\dfrac{17+11}{2}=14$

Jul 6-8:59 AM

How to determine minimum Sample Size needed to Construct confidence Interval

2) Mean

$E = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ with Some algebra work

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2$$

when decimal ⇒ Round-up

If σ is unknown, use S instead

$$n = \left(\frac{Z_{\alpha/2} \cdot S}{E} \right)^2$$

Jul 6-7:49 AM

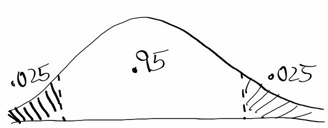
Find minimum Sample Size needed to construct 95% Conf. interval for pop. mean with $\sigma = 25$ and error not to exceed 8.

$\sigma = 25$

$E = 8$

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left(\frac{1.960 \cdot 25}{8} \right)^2 = 37.516$$

$\boxed{n = 38}$

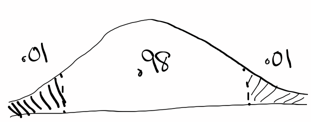


$Z_{\alpha/2} = \text{invNorm}(.975, 0, 1) = \boxed{1.960}$

Redo with 98% C-level and $E = 5$

$$n = \left(\frac{Z_{\alpha/2} \cdot \sigma}{E} \right)^2 = \left(\frac{2.326 \cdot 25}{5} \right)^2 = 135.2569$$

$\boxed{n = 136}$



$Z_{\alpha/2} = \text{invNorm}(.99, 0, 1) = \boxed{2.326}$

Jul 6-9:09 AM

15 randomly Selected teachers from LAUSD had a mean Salary of \$6850/mo. with Standard deviation of \$675/mo.

$n=15$, $\bar{x}=6850$, $S=675$

Find Conf. interval for mean monthly Salary of all teachers in LAUSD.

$\boxed{6476<\mu<7224}$

→ NO C-level → .95

σ unknown → T Interval

$\bar{x}$ → whole # → Round to whole

$E=\dfrac{7224-6476}{2}=\boxed{374}$

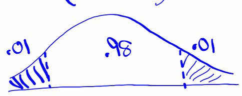
Find min. Sample Size needed if we wish to be 98% confident and error not to exceed \$250.

Since σ is unknown, Use S

$n=\left(\dfrac{Z_{\alpha/2}\cdot\sigma}{E}\right)^2$

$n=\left(\dfrac{Z_{\alpha/2}\cdot S}{E}\right)^2=\left(\dfrac{2.326\cdot 675}{250}\right)^2$

$=39.441$



$\boxed{n=40}$

$Z_{\alpha/2}=\text{invNorm}(.99,0,1)=\boxed{2.326}$

Jul 6-9:18 AM

t - Distribution

It is similar To Z-Dist. but σ is Unknown

1) Symmetric, bell-Shape graph, Total area=1

2) Mean = Mode = Median

3) $\mu=0$ but σ Unknown

It comes with degrees of freedom (df)

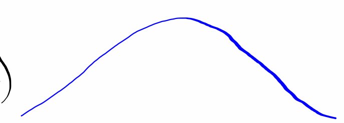
Use

tcdf(L, U, df)

or

invT(Left area, df)

2nd VARS



$\mu=0$

σ Unknown

If You don't have it, download recommended App in Your Smart device.

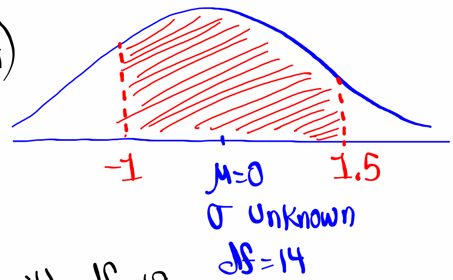
Jul 6-10:01 AM

Find $P(-1 < t < 1.5)$ with $df = 14$.

2nd VARS

tcdf(−1, 1.5, 14)

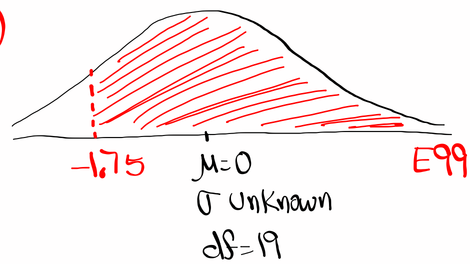
$= \boxed{.755}$



Find $P(t > -1.75)$ with $df = 19$.

tcdf(−1.75, E99, 19)

$= \boxed{.952}$

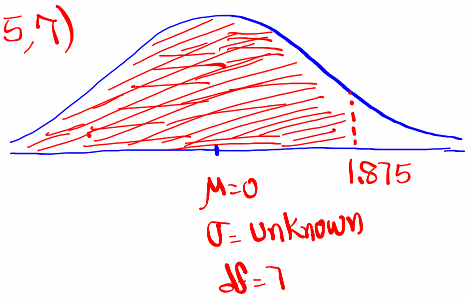


Jul 6-10:08 AM

Find $P(t < 1.875)$ with $df = 7$.

= tcdf(−E99, 1.875, 7)

$= \boxed{.949}$

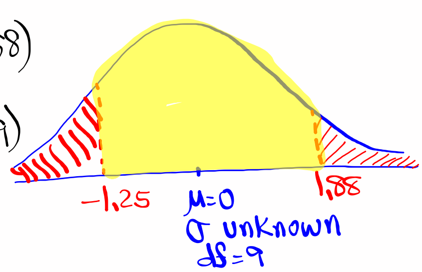


Find $P(t < -1.25 \text{ OR } t > 1.88)$ with $df = 9$.

$= 1 - P(-1.25 < t < 1.88)$

= 1 − tcdf(−1.25, 1.88, 9)

$= \boxed{.168}$



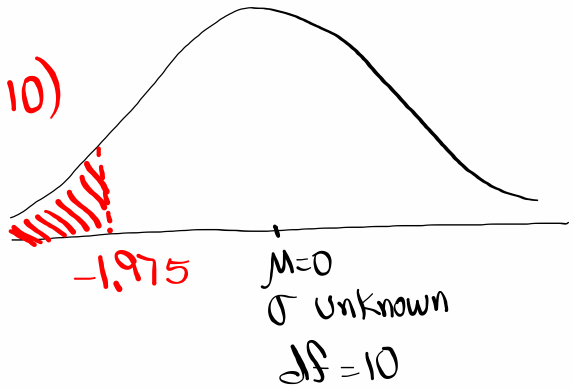
Jul 6-10:13 AM

Find twice the area to the left of

$t = -1.975$ with $df = 10$.

2*tcdf(-E99,-1.975,10)

$= \boxed{.077}$

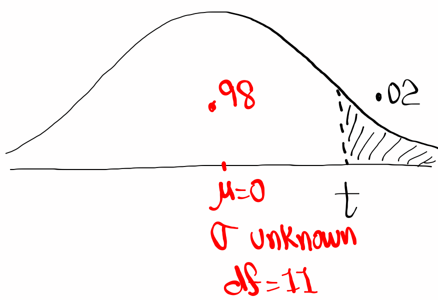


Jul 6-10:19 AM

Find t with $df = 11$ using drawing below

t=invT(.98,11)

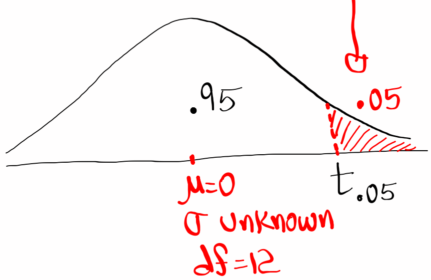
$= \boxed{2.328}$



Find $t_{.05}$ with $df = 12$

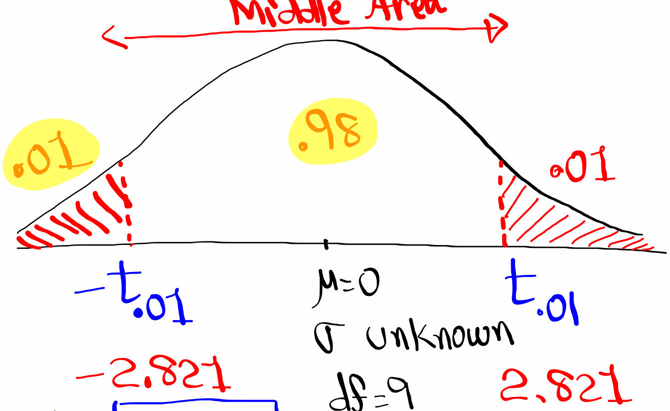
$t_{.05}$ = invT(.95,12)

$= \boxed{1.782}$



Jul 6-10:22 AM

Find $\pm t_{\alpha/2}$ for 98% C-level with df=9.



$t_{.01} = invT(.99, 9) = \boxed{2.821}$

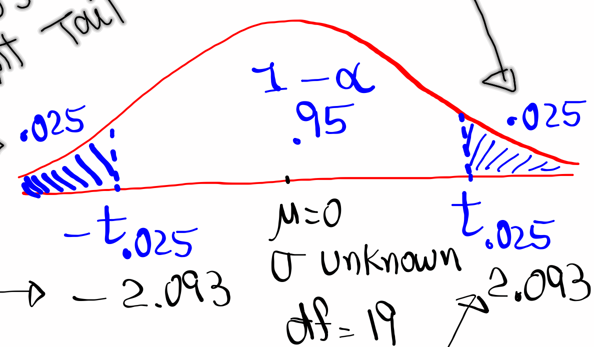
Jul 6-10:27 AM

Find $\pm t_{\alpha/2}$ for $\alpha = .05$ with df=19.

$\alpha/2 = .025$

$t_{\alpha/2}$ Area of right Tail

$-t_{\alpha/2}$ Left Area



invT(.025,19) → −2.093

invT(.975,19) → 2.093

SG 22 & SG 23

Jul 6-10:31 AM

Chi-Square Dist.

$\underbrace{\text{Chi-Square}}_{\chi^2\text{-Dist.}}$

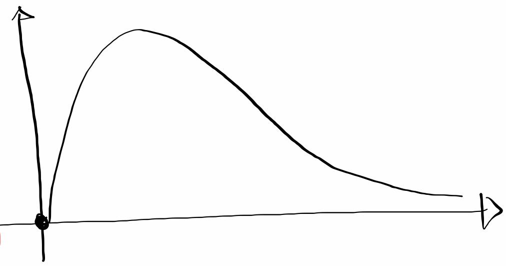
1) Graph begins at 0, and it is skewed to the right.

2) Not symmetric, total area $= 1$

3) It comes with degrees of freedom

Use $\boxed{\text{2nd}}$ $\boxed{\text{VARS}}$

$\chi^2\text{cdf}(L, U, df)$

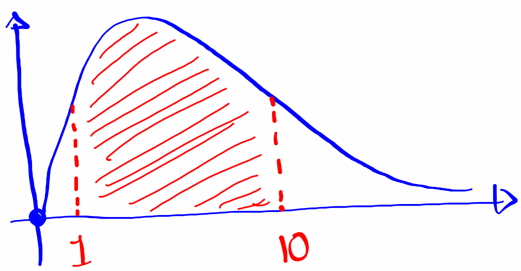


Jul 6-11:02 AM

Find $P(1 < \chi^2 < 10)$ with $df = 8$.

$= \chi^2\text{cdf}(1, 10, 8)$

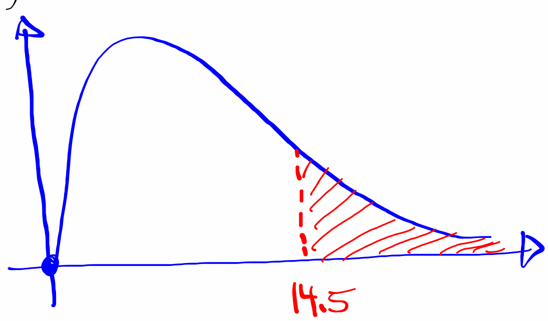
$= \boxed{.733}$



Find $P(\chi^2 > 14.5)$ with $df = 9$.

$= \chi^2\text{cdf}(14.5, \text{E}99, 9)$

$= \boxed{.106}$

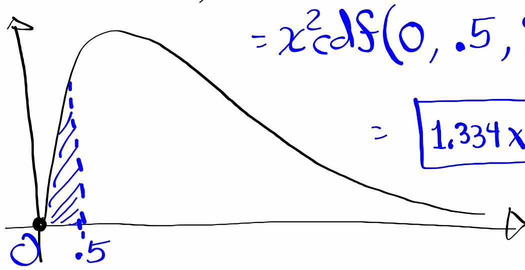


Jul 6-11:05 AM

Find $P(\chi^2 < .5)$ with $df = 8$.

$= \chi^2 cdf(0, .5, 8)$

$= \boxed{1.334 \times 10^{-4}}$

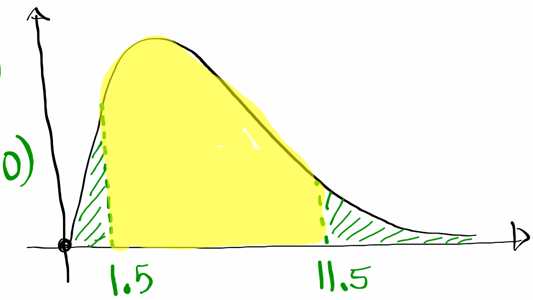


Find $P(\chi^2 < 1.5$ OR $\chi^2 > 11.5)$ with $df = 10$

$= 1 - P(1.5 < \chi^2 < 11.5)$

$= 1 - \chi^2 cdf(1.5, 11.5, 10)$

$= \boxed{.321}$



Jul 6-11:09 AM

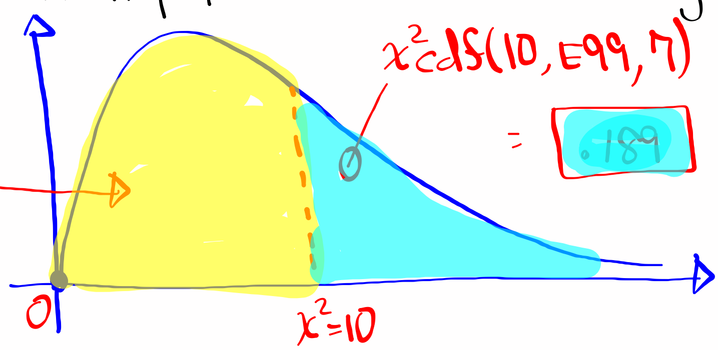
Find the area on each side of $\chi^2 = 10$ with $df = 7$, multiply the smaller area by 2.

$\chi^2 cdf(0, 10, 7)$

$= \boxed{.811}$

$\chi^2 cdf(10, E99, 7)$

$= \boxed{.189}$



$2(\text{Smaller Area}) = 2(.189) = \boxed{.378}$

Jul 6-11:15 AM

what is degrees of freedom?

7 of you

I bring 7 donuts.

Adrian	7	choices
Ashley	6	"
Christian	5	"
⋮	⋮	⋮
Last person → Montserrat	0	" (1 Donut left)

$df = 7-1$
$= \boxed{6}$

Maggie has 5 shirts to use to go to work.

Monday	5	choices
Tuesday	4	"
Wed.	3	"
Friday	0	" (1 shirt left)

$df = 5-1$
$= \boxed{4}$

Jul 6-11:21 AM